TILE PATTERN CLASSIFICATION – A STATISTICAL ANALYSIS

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ABSTRACT

This article presents a mathematical analysis of the tile patterns included in the *Az Infinitum – Azulejo Indexation and Referencing System*. Based on previous experiences described in the literature, the authors catalogued 510 patterns to obtain some empirical data on the frequency of the different types of tile patterns, aiming at contributing to a more comprehensive knowledge of these patterns.

KEYWORDS

Azulejo | **Glazed ceramic tiles** | **Mathematics** | **Patterns**

INTRODUCTION

The *azulejo* (glazed ceramic tile) is, for many, a Portuguese national symbol. What is less well known is that, beyond creativity and history, mathematics can also be found in these ceramic tiles.

This association between mathematics and tile patterns has already been the subject of several articles. One of them, for example, examines the tiled façades of the city of Ovar (Atractor, 2018), other focuses on tiles with two colours (Hall et al., 2023), and other studies some tile panels in public buildings on the south bank of the Tagus (Rodrigues, Freitas, 2018). In these articles, the patterns found on the tiles are categorised according to their symmetries, i.e. the transformations of the plane that leave the pattern invariant. The word "symmetry" usually refers to mirror reflections, but in maths, rotational and translational symmetries are also considered. For example, a pattern formed with a simple square motif, following the edges of the tile, has reflection symmetries in relation to the square's medians and diagonals, rotation symmetries of 90°, 180° and 270° around the centre of the square, and translation symmetries in two directions, horizontal and vertical (considering the pattern here as an infinite paving of the entire plane).

These sets of symmetries have been studied for regular coverings of the plane, and it has been proven that there are only 17 possible sets of symmetries. Each of these possibilities is described by the Washburne and Crowe classification[1]. This classification characterises patterns according to their symmetries, assigning each one a code with 2 to 4 alphanumeric characters. Although we do not want to go into a very detailed description of these codes, we give an idea of what some of these characters mean.

- The code starts with **p** or **c**, depending on how the pattern is reproduced (this letter related to the directions in which the pattern is reproduced).
- The numbers have to do with the angles of rotations: number **1** indicates that there are no rotations that preserve the pattern, number **2** indicates that there is a 180° rotation, number **4** refers to 90° rotations (other numbers can also be used).
- The following characters (if any) can be **m**, which is a reference to the presence of mirror reflections, or **g**, which refers to a reflection followed by a translation parallel to the axis of reflection (called a *glide reflection*).

The aforementioned articles classify a number of tile patterns in a certain geographical region, sometimes also restricted to a time period, thus contributing to the study of the taste reflected in the patterns in that area and at that time, adding a mathematical element. The main conclusion obtained was that a small minority of the classifications were sufficient to characterise the majority of the tile patterns analysed, namely p1 (without symmetries except the ones from translations), p4 (only with 90° rotation symmetry) and p4m (with 90° rotation symmetry and at least 2 reflection symmetries that make 45° between them).

In this article, we carry out a similar study, obtaining some empirical data on the frequency of the different types of tile patterns found in the large database of the *Az Infinitum – Azulejo Indexation and Referencing System*, an online platform, constantly updated, which inventories, catalogues and documents the tile heritage produced or applied in Portugal to date. From this, we have only chosen pattern type tiles, i.e. those whose identifier begins with 'P-' followed by a number (we recommend accessing the site to find out more information about a specific pattern). With this restriction, we evaluated 510 different tile patterns produced or otherwise applied in Portugal.

1. Originally published in (Washburn, Crowe, 1988), it can also be found in (Veloso, 2012).

CLASSIFICATION

Although Washburn and Crowe's classification is clear, the classification of the tiles is not as objective as one might like, so some rules had to be established from the outset to try to keep the analysis of the data as close to the materiality of the tile as possible. We therefore assigned each tile two classifications, which we have called strict and approximate, which we will describe below.

HUMAN ERROR AND ARTISTIC DECISION

The classification we call strict uses a rigorous mathematical view to analyse symmetries. Since many tiles are handmade, they tend to have small imperfections in the symmetries. Also, their exposure to the elements can lead to some symmetries to be lost over time. As such, this explicitly mathematical classification means that the vast majority of patterns would be classified as p1 (without symmetries except the ones from translations). In the approximate classification, we accept faults in the symmetry that are the source of human error or are undetectable to an observer analysing the tile without further tools, thus considering these errors to be negligible [fig.01,02,03].

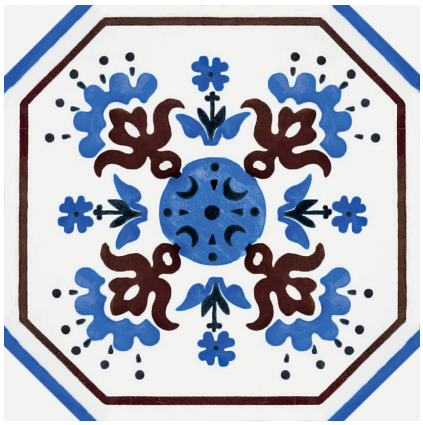


Fig. 01. Pattern P-19-00142: The positions of the small dots at the top of each flower invalidate the symmetries in the strict classification (source: *Az Infinitum*, designed by Inês Aguiar)

Fig. 02. Pattern P-19-00150: The green areas are not evenly coloured, which also invalidates the symmetries in the strict classification (source: *Az Infinitum*, designed by Inês Aguiar)

Fig. 03. Pattern P-20-00132: The four flowers in the centre are arranged in such a way that the pattern has no rotational symmetry, although this is clearly intended (source: *Az Infinitum*, designed by Inês Aguiar)

SENSE OF THE SYMMETRY

The use of the approximate classification needs some justification due to the additional subjectivity it introduces, on top of the one strict classification already entails. A secondary objective of this work is to identify common patterns in tiles and try to extrapolate conclusions about which ones might be more pleasing to the human experience, within a certain cultural context. For this purpose, having results that are presented as p1, or without symmetries (other than translational), when they are easily identified as having 'quasi-symmetries', i.e. some visual harmony identifiable with a group of patterns with the same classification, makes no sense, since the object of study is inevitably artistic.

OTHER NOTES ON CLASSIFICATION

Tilings can be obtained from a *motif*, that is, a small, limited figure that generates the whole tiling by translations. Usually, the square tile can be taken as a motif, as the pattern consists of translations of this tile, with no rotations. This is called a *module* in the Az Infinitum database.

It should also be noted that, in some instances, the *Az Infinitum* database only shows a *reduced motif* as the motif of the pattern – in other words, to obtain the desired plane tiling, you must resort not only to translations, but also to rotations or reflections of this reduced motif. When the tile itself is used as a reduced motif (and is rotated, for instance, to create the motif for the tiling), the larger motif created is called a *composite motif* (Rodrigues, Freitas, 2018). There are thus several ways to use the reduced motif to produce different motifs and patterns [fig.04]. In the cases where a composite motif and/or multiple motifs are presented in the database, strictly speaking, the first option presented was chosen. In any case, these cases are a much smaller number than the total of patterns analysed, so they won't affect the conclusions.

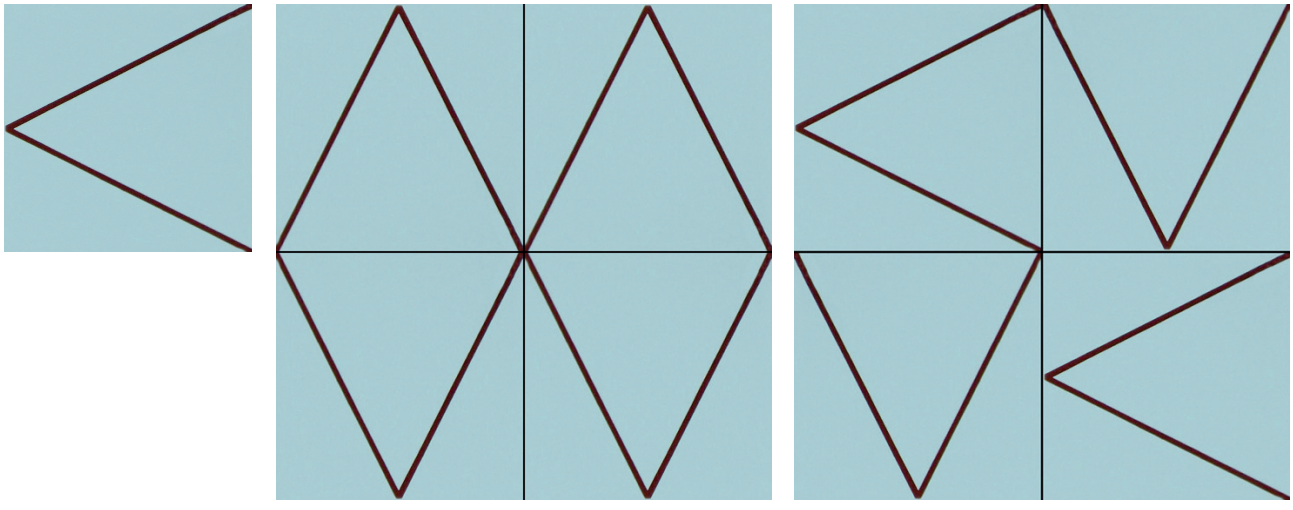
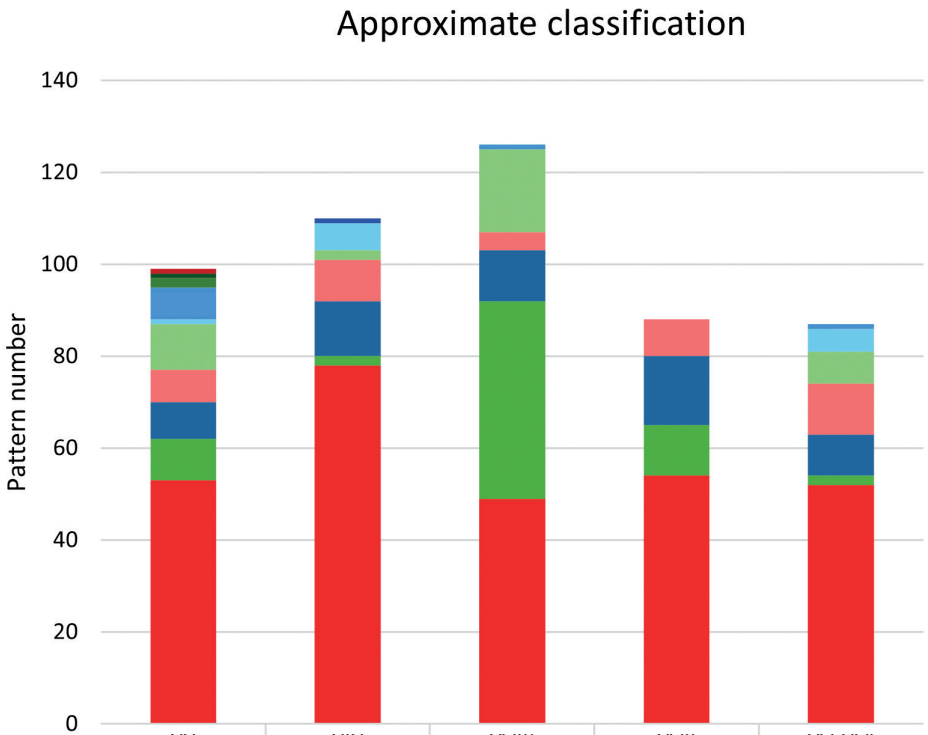


Fig. 04. Pattern P-20-00048: a tile as reduced motif, with two possible composite motifs (source: Az Infinitum, designed by Inês Aguiar)

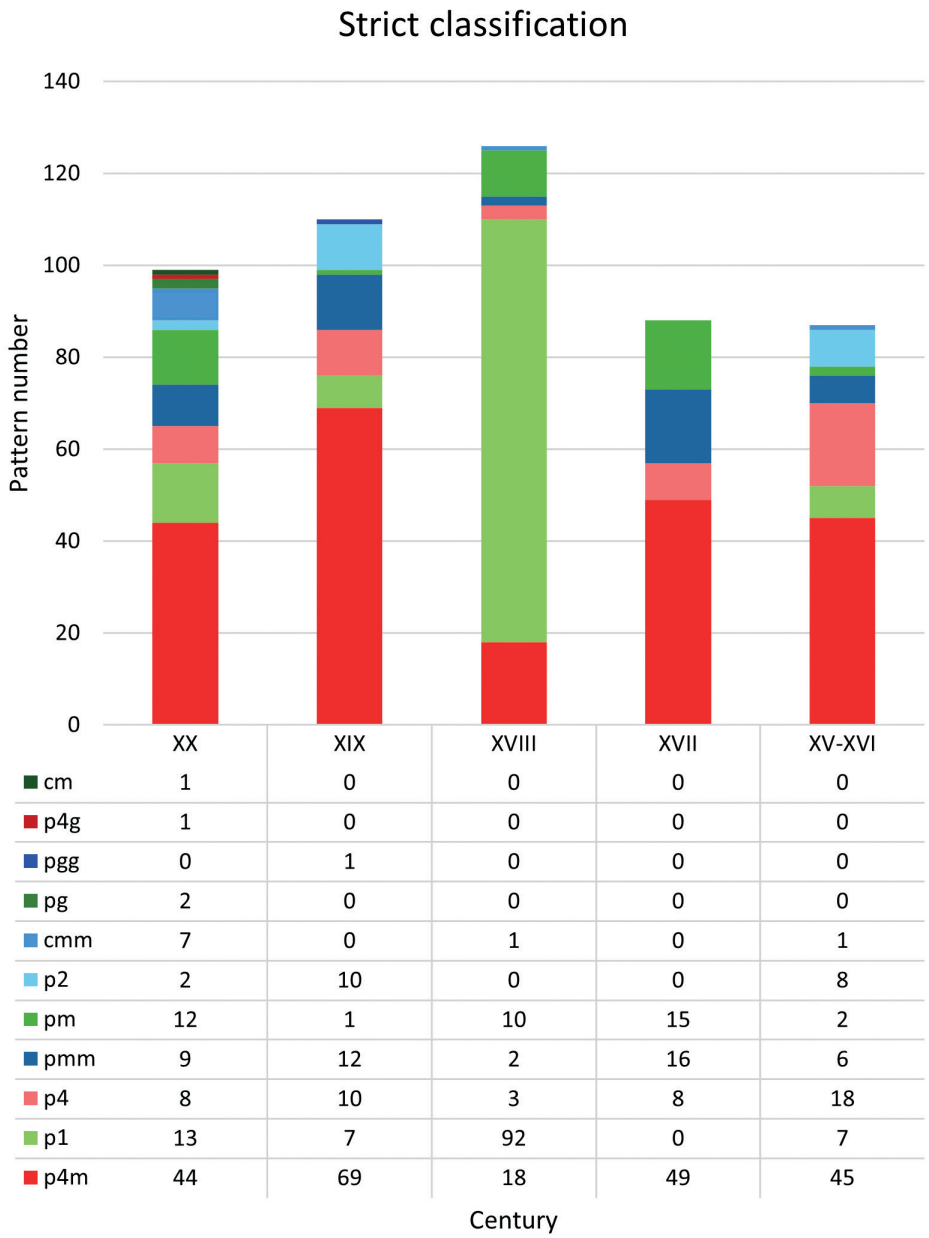
RESULTS AND ANALYSIS

We present here the statistical results obtained for the approximate and strict classifications, respectively, adding the division into centuries of these classifications [graph.01,02].

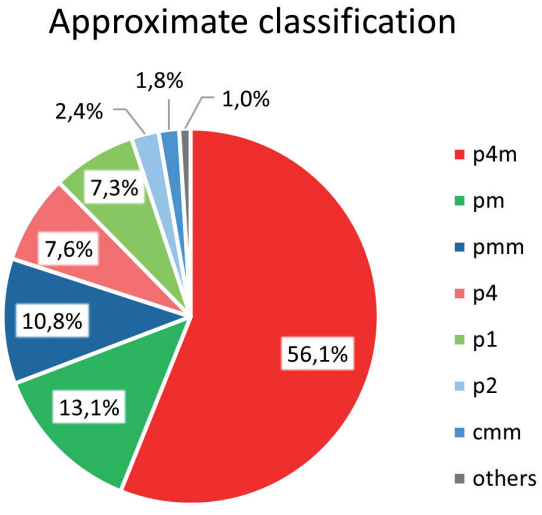
We can immediately conclude from the circular graphs [graph.01] that what was found by Rodrigues and Freitas is indeed true (Rodrigues, Freitas, 2018): the most frequent classifications are p1, p4 and p4m, accounting for more than 70% of the patterns studied in both analyses. The classifications pm (only one reflection symmetry) and pmm (180° rotation symmetry and reflections in two directions) also have a frequency comparable to p4 or p1. We can also see that, as expected, p1 is much more recurrent in the strict classification than in the approximate one.

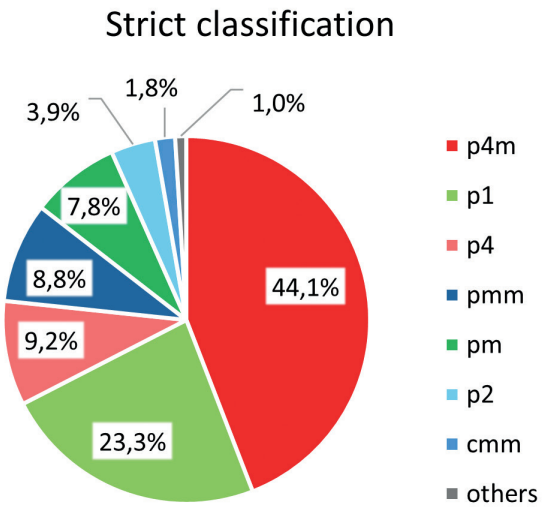


	XX	XIX	XVIII	XVII	XV-XVI
pgg	0	1	0	0	0
p4g	1	0	0	0	0
cm	1	0	0	0	0
pg	2	0	0	0	0
cmm	7	0	1	0	1
p2	1	6	0	0	5
p1	10	2	18	0	7
p4	7	9	4	8	11
pmm	8	12	11	15	9
pm	9	2	43	11	2
p4m	53	78	49	54	52

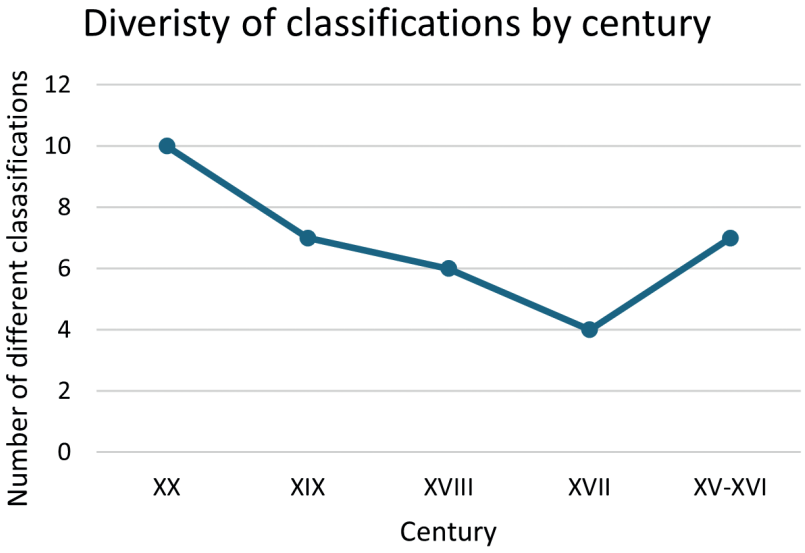


	XX	XIX	XVIII	XVII	XV-XVI
cm	1	0	0	0	0
p4g	1	0	0	0	0
pgg	0	1	0	0	0
pg	2	0	0	0	0
cmm	7	0	1	0	1
p2	2	10	0	0	8
pm	12	1	10	15	2
pmm	9	12	2	16	6
p4	8	10	3	8	18
p1	13	7	92	0	7
p4m	44	69	18	49	45





Graphs 01. Graphic representation of the statistical analysis of the gathered data of the tiles in the database in which the red, purple, blue and green colors are associated with rotation symmetries of (respectively) 90°, 120°, 180° and 360° (source: authors)



Graph 02. Graphic of the number of different patterns by century (same values for both the strict and approximate classifications) (source: authors)

POMBALINE STYLE

It is interesting to discuss why there are so many patterns classified as p1 in the 18th century tile patterns, as is apparent from the bar graphs [graph.01]. The database used has an abundance of tiles labelled "Pombaline" in the catalogued 18th century tiles. Most of these tiles have a common property: they seek to simulate the incidence of light (regularly from above) which means that, for example, in patterns with floral themes in the corners of the motif, these are coloured with different intensities and sometimes different colours in different corners [fig.05]. These variations come from artistic choices, so most of the patterns in this style have been categorised as p1 in the strict classification, but differently with the approximate classification.

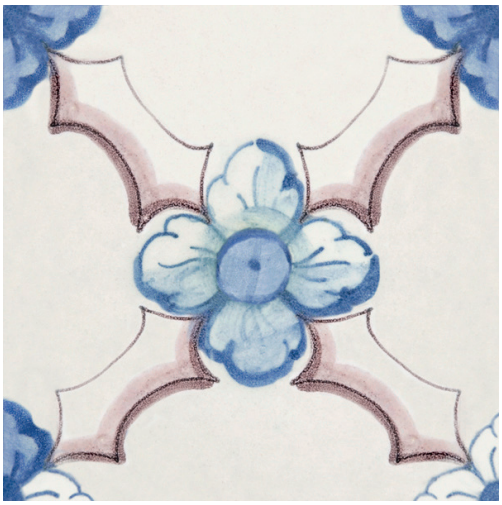


Fig. 05. Pattern P-18-00116: Pombaline pattern "simulating the incidence of light, orientated from above, suggesting volumetry" (source: *Az Infinitum*, designed by Inês Aguiar)

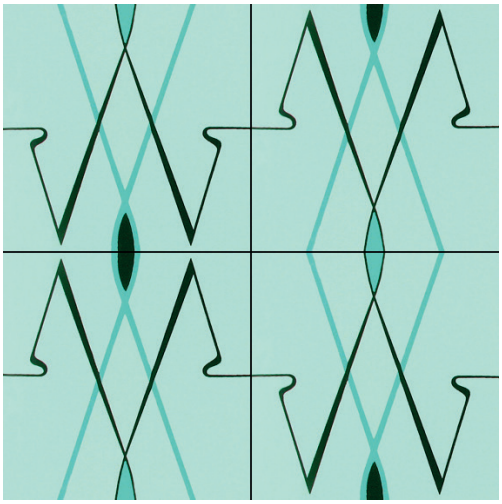


Fig. 06. Pattern P-20-00050: 20th century pattern with classification cmm (source: *Az Infinitum*, designed by Inês Aguiar)

An interesting conclusion is that the 20th century clearly has the greatest diversity of patterns [fig.06]. This is to be expected, given that it was in the 20th century that the modernist movement occurred, bringing about a subversion of artistic traditions (this database does not currently include patterned tiles from the 21st century, so it is plausible that there is even greater diversity there).

AESTHETICS

Comparing the results obtained with the literature on human aesthetic preference of patterns, we see that the results seem to be in line with what is expected. Reham Sanad shows the following order of pattern preference is presented (Sanad, 2017), obtained from a study of more than 800 pattern evaluations:

p4m > p6 > p3m1 > p6m > p4 > p31m > cmm > p4g > pmg > pmm > p3 > pm > p2 > cm > pg > p1 > pgg

To compare this result with the one we obtained, it is important to bear in mind that all the tiles analysed are square, and many of the patterns that appear to be preferred in this study don't work so well on square tiles. In particular, patterns with rotational symmetries of 60° and 120° do not adjust to square tiles. These are classified as p6, p3m1, p6m, p31m and p3. Some patterns are, however, quite close: in figure 7 can be seen a vertical dilation of a p3m1 pattern (it would be such a pattern if the tile had proportions 1 to $\sqrt{3}/2$ and the triangles were equilateral) [fig.07].

We present the order below, after removing the classifications referred to above from the original order in (Sanad, 2017), followed by the order obtained here through our approximate classification[2]:

p4m > p4 > cmm > p4g > pmg > pmm > pm > p2 > cm > pg > p1 > pgg

p4m > pm > pmm > p4 > p1 > p2 > cmm > pg > cm > p4g > pgg > pmg

2. We excluded the only classification with a rotation symmetry other than 90 or 180 for comparison, for the reasons already given, and added the pmg classification in last place to include all the classifications in the order present in the referenced article.

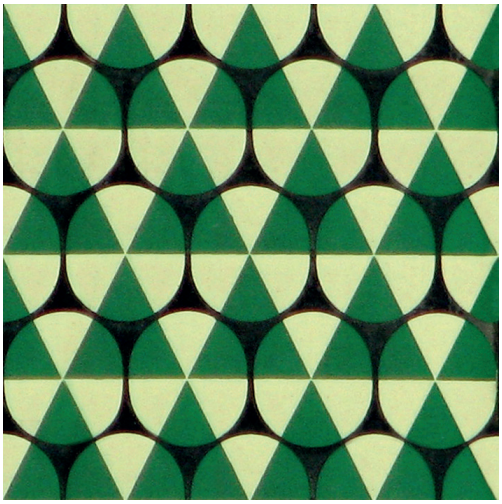


Fig. 07. Pattern P-20-00038: A pattern in the database which is a small dilation of a p3m1 pattern (source: *Az Infinitum*, designed by Inês Aguiar)

Comparing these two lists, we can immediately conclude that there is a clear preference for p4m patterns. Figure 8 shows two examples of tiles that generate patterns with this classification: it has eight mirror symmetries, along the medians and the diagonals pf the tile, and four rotational symmetries (including the identity) [fig.08]. The pm and pmm classifications appear more frequently than expected. This may be because, despite the use of approximate classification, many patterns that could be aesthetically assessed as p4 or p4m are still being classified as pm or pmm because the differences are significant. The same explanation can be attributed to p1 patterns.

It should also be noted that aesthetic preference is not the only decisive force in the creation of tile patterns. Like any piece of art, many of the patterns seek, for example, to subvert the expectations of what was previously created, sometimes even to the detriment of immediate aesthetic preference. So, we can also attribute some of the discrepancies to this.

Fig. 08. Examples of tiles with the most frequent classification, p4m (source: *Az Infinitum*, designed by Inês Aguiar)

Finally, we note that this study was carried out in 2017, so it reflects the aesthetic preferences of the 21st century, which can differ greatly from those of previous centuries. For example, the cmm pattern appears in a fairly high position in the aforementioned study. Also, in the 20th century data in the bar charts, we see that cmm has a significant fraction in the 20th century, but less in previous ones.

This last section shows how difficult it is to carry out an impartial study when classifying something as human as art, which is inevitably linked to those who analyse it.

AUTOMATIC CLASSIFICATION

There is a possible way to mitigate this bias. While in this work the patterns were identified manually, algorithms have been developed and are constantly being updated (Reddy, 2005) that seek to automate this process (Collins, 2004). It seems to be an interesting idea, perhaps in the interest of the *Az Infinitum* team, to try to collaborate with those responsible for this computational model to include an automated classification on their website, which nevertheless takes into account the small imperfections and artistic choices we have mentioned.

CONCLUSIONS

To summarise, it was found that the most common pattern in this database is clearly p4m followed by p4, p1, pm and pmm, in no particular order, and that this seems to make sense with social studies regarding human aesthetic preference. There is an open possibility for the application of algorithmic methods to the study of patterns in tiles, especially as it turns out that manual classification is amenable to great impartiality, although some attention is also needed to possible lacks of strict symmetry that are merely a consequence of the artisanal nature of manufacture. In general, this area still has a lot to study.

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