TILES AND IDENTITY BY PATTERN CLASSIFICATION

IDENTIDADE E AZULEJOS PELA CLASSIFICAÇÃO DE PADRÕES

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ABSTRACT

Walled tiles can be figurative or patterned. Whereas the figurative tiles can better be described by theme or author, tile patterns are traditionally classified using more abstract rules that describe either the motif or the pattern itself. In this paper, we present a traditional mathematical classification of plane patterns, the Washburn and Crowe Algorithm, and use it to identify or distinguish tile patterns. We present a complete mathematical classification of the tile patterns present in all places of public access in the Almada region and show how this classification can help recover damaged tiled walls and floors, in order to preserve our heritage. We extend this mathematical analysis to 20th century patterns and quasipatterns, hoping to show that this classification can add to our knowledge of the identity of these patterns.

KEYWORDS

Classification | **Tile patterns** | **Mathematics** | **Symmetries**

RESUMO

Os revestimentos azulejares podem ser figurativos ou de padrão. Enquanto os painéis figurativos podem ser melhor descritos por associação a um tema ou a um autor, os padrões de azulejos são tradicionalmente classificados usando regras mais abstratas que descrevem o motivo ou o próprio padrão. Neste artigo, apresentamos uma classificação matemática tradicional de padrões no plano, o algoritmo de Washburn e Crowe, e utilizamo-lo para identificar ou distinguir padrões de azulejos. Apresentamos uma classificação matemática completa dos padrões de azulejos presentes em todos os locais públicos ou de acesso público na região de Almada e mostramos como essa classificação poderá auxiliar na recuperação de paredes e pisos com azulejos danificados, a fim de preservar o nosso património. Estendemos essa análise matemática a padrões e quase-padrões do século XX, na esperança de mostrar que essa classificação pode contribuir para o nosso conhecimento da identidade desses padrões.

PALAVRAS-CHAVE

Classificação | **Padrões de azulejos** | **Matemática** | **Simetrias**

INTRODUCTION

After visiting the famous Alhambra Palace, M. C. Escher (1898-1972) devoted himself to the study of patterns which could be obtained from the careful repetition of a certain motif. He came to learn that there are seventeen different types of plane patterns and seven different types of frames, all seen in this magnificent palace[1].

Is there more diversity? In artistic terms, of course there is, in mathematical terms, it is proved that there is not. There are several algorithms, such as the one formulated by Washburn and Crowe (Crowe and Washburn, 1988), which allow us to classify into one of those seventeen or seven types any plane pattern or frame, according to the geometric transformations it displays: translations, rotations, reflections and glide reflections.

In Portugal, despite the diversity of patterns that can be observed in tiles, namely from the 14th to the 19th centuries, there seems to be no complete representation of the seventeen types of mathematical plane patterns. The very shape of most tiles, square or rectangular, prevents the appearance of some types of patterns, which would require triangular or hexagonal tiles. In addition, there are patterns that, despite being visually distinct, end up having the same mathematical classification. For instance, the patterns in **Fig. 01** have the same classification, since the only geometric transformations that keep them unchanged are rotations of 90 degrees (and multiples of 90 degrees) and translations in two directions. The second and third patterns have no reflections that would keep them unchanged, due to the interlacing observed. The first pattern has no reflections because the interior of the two kinds of leaves is different. This pattern tiles can be observed in the Capuchos Convent, in Almada.

Fig. 01 · Almada, Capuchos Convent. Distinct tile patterns with the same classification (photos by Fátima Rodrigues)

There are already some well-known studies of this type. In the bibliography related to Portuguese tiles, the 1st volume of the 4th tome of the monumental work "Corpus da Azulejaria Portuguesa" (Simões, 1971) presents a classification of the patterns based on the tiles that are used. This has the same intention as the mathematical classification, even if it distinguishes patterns which are mathematically the same. This classification involves associating a code to each tile and to each pattern formed with this tile. The study also links this information with the place where the tile pattern is located. A similar type of classification is done in the

1. In the context of the study of tile patterns, the word "frame" is the one that describes, in the most general terms, a pattern that is repeated in only one direction. In the context of pattern classification, the word usually used is "frieze", but in the context of tile patterns, this word has a more strict meaning.

Az Infinitum database[2], which we will use later in this paper. As we have noticed, these classifications have the same intention as the mathematical classification, but they go a little further: some patterns which are distinct in these classifications may have the same mathematical classification, as we will see.

CASE STUDY 1

In 2013, there was a study carried out within the MATER exhibition (FCT NOVA)[3], which surveyed the patterns on the south side of the Tagus river, namely in the Capuchos Convent, S. Paulo Seminary, Igreja do Monte, Casa da Cerca and Solar dos Zagallos.

The Director of the Museum of the City of Almada provided the mapping of the tile heritage and the public tile art of the region. In the study, an exhaustive analysis and a photographic report of all the patterns and frames, made of different materials, including tiles, was carried out. A script with the classification of the frames and patterns existing in all places of public access was produced. In a panel of the MATER exhibition the result of the study was portrayed, including a photographic representation of each of the places above mentioned and all the respective patterns and frames that can be found in such locations, appropriately classified.

As a result of that extensive study we point out that only a total of 9 mathematical types of patterns and 6 mathematical types of frames were identified, considering all types of materials. However, if we only consider the patterns present in tiles, just 3 mathematical types were identified: those unchanged only by translation (p1), those unchanged by translation and 90-degree rotation (p4), and those that, besides the already mentioned geometric transformations, are also unchanged by reflections whose axis draw a 45-degree angle (p4m).

Moreover, no tile patterns were found at Igreja do Monte and Casa da Cerca. At Capuchos Convent, four tile patterns were found, all of the same type, p4. At S. Paulo Seminary, there are six patterns of type p4m and one of type p1, and Solar dos Zagallos contains one pattern of each type: p1, p4 and p4m.

Since the tiles are old and were handmade, we must ignore some imperfections that may occur, otherwise we would never find a pattern. However, the details of color or design applied by the artist must be considered in the mathematical classification. That is the case of the above-mentioned patterns in **Fig. 01**, from Capuchos Convent. It is also the case of the three patterns of Solar dos Zagallos **(Fig. 02)** which have different mathematical classifications despite seeming to remain unchanged by the same geometric transformations.

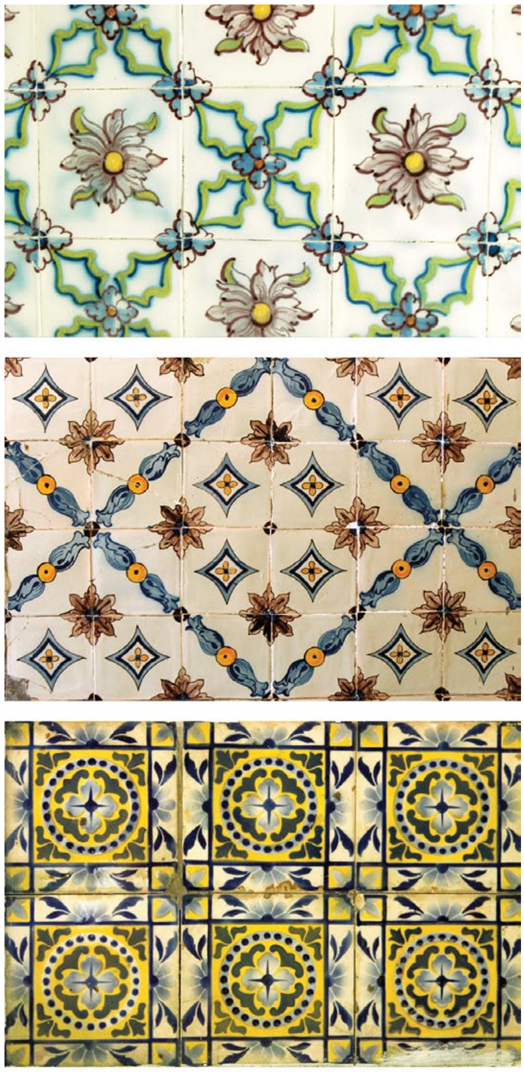

Fig. 02· Almada, Solar dos Zagallos. Tile patterns p1, p4 and p4m (photos by Fátima Rodrigues)

2. Azulejo Indexation and Referencing System. Available at http://redeazulejo.fl.ul.pt/pesquisa-az (2018.05.28).
3. MATER exhibit – panels "A Matemática na Arte", Building VII - FCT NOVA (since 2013)

For the first pattern (p1), color details prevent the existence of reflections that would keep the pattern unchanged. In the second pattern (p4), the same occurs but with details of design. Only the third pattern (p4m) is kept unchanged by reflections (with respect to the diagonals of the tile and to the lines parallel to the sides of the tile that go through the center).

Regarding the frames, out of the seven types of mathematical classification, only one was not found. That type of frame is the one that remains unchanged by the following geometric transformations: translation, reflection in a vertical axis, glide reflection and 180-degree rotation (pma2).

As for the distribution of the various types of frames between the locations, we found that: at S. Paulo Seminary, there are twenty frames of five types; at Casa da Cerca and Solar dos Zagallos four and five respectively, of four different types; at Igreja do Monte only three frames each of a different type; and at Capuchos Convent we find six frames of the same type and two of another.

It would be interesting if a study were to be conducted about the relationship between the age of each tile pattern and frame and their complexity, regarding both color and design. For example, at Capuchos Convent there is only one type of pattern, which has small details in the design of each tile that prevents reflections. It is also at Capuchos Convent where we find the least diversity of frames, with only two types present. Both have translations and reflections in vertical axis, but one has a reflection in the horizontal line in the middle of the frame and rotation of 180 degrees (pmm2), while the other does not have horizontal nor glide reflection, nor 180-degree rotation (pm11).

CASE STUDY 2

For a second panel of the MATER exhibition another kind of mathematical study was conducted, based on fragments of tiles from the 16th century coming from Palácio Nacional de Sintra, Mosteiro de Santa Clara-a-Velha, in Coimbra, and Museu do Teatro Romano, in Lisbon, all of Hispano-Moresque origin. These tiles were being investigated by Susana Coentro from VICARTE, in particular, their chemical properties and the analysis of their materials.

The Hispano-Moresque ceramic tiles were the first to be used on a large scale in Portugal. They appear in religious buildings such as churches, convents and seminaries, but also in palaces. The vast diversity of patterns is related to the evolution of the production techniques which took place from the 14th-16th centuries. The evolution of tiling techniques (*alicatado* technique, *cuerda-seca*, *arista* tiles) followed the evolution of the society tastes, from the typically Islamic patterns to figurative motifs, such as vegetal and anthropomorphic patterns. By the mid-16th century, these techniques were no longer used. They were replaced by a new technique: the *majolica* tile. Hispano-Moresque ceramic tiles from the 15th-16th centuries show a wide range of decorative patterns, essentially geometric, as told by Susana Coentro (2017) in her PhD thesis "An Iberian Heritage: Hispano-Moresque Architectural Tiles in Portuguese and Spanish Collection".

The study of the patterns can be associated to other studies, namely those of the chemical identification of glazes and ceramic pastes of the tiles, in order to better characterize the tastes of a certain time or place, or even the provenance of each tile, through the materials used to produce it.

This mathematical study can also apply to heritage conservation, related to the work of recovering tile patterns, walls and floors of monuments in which the original is degraded. From a piece of broken tile, one can reproduce the tile and, through the various geometric transformations, arrive at a type of mathematical pattern that is identified with the artistic pattern.

The authors believe that this classification of patterns can also contribute to the determination of the historical and geographical identity of pattern tiles, providing a means of bringing together or telling apart patterns and tiles and highlighting which ones are more frequent. Exactly because one pattern may be visually different from another and have the same mathematical classification, this study may provide a newfound, less visible, proximity.

Fig. 03· Coimbra, Santa Clara-a-velha Monastery, artistic pattern produced from a tile fragment (photo by Susana Coentro and pattern design by Fernanda Barroso)

In this mathematical study, we considered the highest number of fragments with an identifiable design and proceeded to create the patterns based on a fragment, after drawing the stylized tile, as illustrated in **Fig. 03**.

Analyzing the patterns produced, a very reduced number of mathematical patterns can be found: p1, p2, p4, p4m and pmm. The pattern of type p2 is the one in **Fig. 03**, where we find 180-degree rotations (and its multiples) in addition to the translations in two distinct directions. Two patterns of type pmm were obtained from two tile fragments from Santa Clara-a-Velha. In these, besides the 180-degree rotations and translations in two directions, there are also reflections in axis with distinct directions and all the centers of rotation are on the reflection axis.

The patterns produced and their respective classification are part of a panel in the exhibit "MATER – A Matemática na Arte", produced by FCT NOVA.

Comparing both case studies, the diversity of patterns produced based on tile fragments – a total of five – is higher than the diversity of patterns found in the places of the Almada region – only three.

MATHEMATICAL CLASSIFICATION

Without going into too much detail, we will say that this mathematical classification of the patterns depends on the *symmetries* of the tiles used and of the patterns that can be built with them. By symmetry, we mean any transformation of the plane that keeps the pattern unchanged. Most known examples are rotations, reflections, and translations. In this approach, all transformations considered are isometries, that is, transformations which preserve distances and angles within the pattern, and therefore do not deform, increase or reduce the images.

A pattern is a drawing in the plane formed by a composite motif that is repeated periodically in two directions. A frame is originated by a composite motif that is repeated periodically in one direction. The composite motif results from performing one or more isometries on a minimal motif that will be the basis of the entire pattern.

Thus, to create a pattern, we need to make the composite motif and translate it, periodically, in two different directions. With the same composite motif, we can obtain distinct patterns depending on the two directions of the translation chosen, but all of them with the same mathematical classification.

Furthermore, with the same minimal motif we can obtain all the seventeen kinds of patterns and all the seven types of frames, given that every one of them depends on the composite motif that was created. This can be observed in one of the panels of the MATER exhibition, where we can see displayed all types of patterns and frames generated from a stylized "boat sail".

But what isometries can we use to produce the composite motif? By mathematical arguments of linearity, the isometries we can use are translations, rotations (of 60°, 90°, 120° or 180°, by the Crystallographic

Restriction Theorem), reflections and glide reflections (a reflection followed by a translation in the direction of the reflection).

Why seventeen kinds of patterns and seven types of frames? By mathematical arguments of Group Theory, we can prove that there are only seven types of one-dimensional patterns, seventeen types of two-dimensional patterns, and two hundred and thirty types of three-dimensional patterns. See for instance (Perez and Reis, 2002) for a full proof of this classification for the patterns in the plane. We can also prove that the isometries that leave an invariant pattern are the same ones that are used to produce the pattern, although not in a unique way, as we can see in Fig. 04.

In fact, we can produce the composite motif of the Fig. 04 in two different ways: by vertical reflection followed by a rotation of 180° or by vertical reflection followed by a glide reflection.

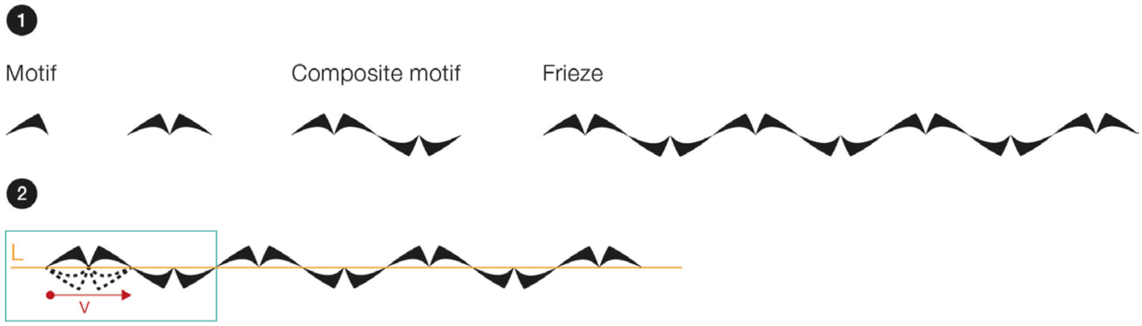

Fig. 04· Mathematical frieze description: two different ways to produce the same frieze (pma2) (design by Camy)

In fact, we could enumerate the isometries that keep each pattern or frame unchanged. However, there is no need to exhaustively observe all isometries in order to classify the pattern, since they are interchangeable in the production of the composite motif, as illustrated in the example above.

These results are the base for algorithms like the Washburn and Crowe Algorithms for frames and patterns, which in an optimized way allow us to classify each frame and pattern as one of the seven or seventeen types respectively.

The Washburn and Crowe Algorithm for frames begins with their division in two types:

a) with vertical reflection
- a.1) with horizontal reflection – pmm2
- a.2) without horizontal reflection
  - a.2.1) with 180-degree rotation – pma2
  - a.2.2) without 180-degree rotation – pm11

b) without vertical reflection
- b.1) with horizontal or glide reflection
  - b.1.1) without horizontal reflection – p1a1
  - b.1.2) with horizontal reflection – p1m1
- b.2) without horizontal nor glide reflection
  - b.1.1) with 180-degree rotation – p112
  - b.1.2) without 180-degree rotation – p111

The Washburn and Crowe Algorithm for patterns begins by separating the patterns in five categories according to their minimal angle of rotation:

a) 60 degrees – p6, p6m
b) 90 degrees – p4, p4m, p4g
c) 120 degrees – p3, p3m1, p31m
d) 180 degrees – p2, pgg, pmg, pmm, cmm

without rotations that keep the pattern unchanged: p1, pg, pm, cm

In this last category, the patterns produced are the same type as Fig. 05.

The name code of each pattern and frame starts with a "p", except for two patterns: cm and cmm. In both cases, the composite motif was obtained through the reflection of a minimal motif, whose reflection axis is not parallel to the directions of the translation. The letter "m" means there are (mirror) reflections, and when repeated it means there are reflections in a non-parallel axis. The letter "g" is for glide reflections and the numbers 2, 3, 4 or 6 stand for the minimal angle of rotations being 360° divided by the respective number. These techniques also allow us to quantitatively distinguish some works of 20th century tiles, in which the uniformity of the pattern is broken, while maintaining symmetries, sometimes with a local, non-global character. We present a few examples in the next two sections.

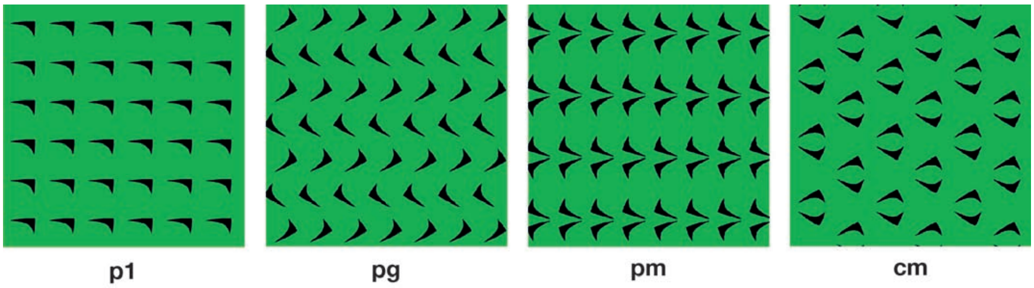

Fig. 05· Examples of pattern types: patterns having no rotation that leaves them unchanged (design by Camy, using software GeCla available at http://www.atractor.pt/soft.html)

COMPOSITE MOTIFS

In this section, we present some recent examples of tilings in which the pattern is preserved, but is achieved either with several different tiles that fit together, or with a single tile, which is placed in different positions in order to form a motif composed of several copies of this same tile.

This type of composite motif appears very frequently in pattern tiles from past centuries (it is not characteristic of the 20^{th} century). **Fig. 01** already presents patterns in which the motif is created by a single tile, which is then rotated to form a 2x2 motif, which then generated the pattern by simple repetition. In (Simões, 1971) one can find a very big variety of such patterns formed by more than one tile. There are several 2x2, 4x4 and 6x6 examples, most of them formed in very much the same way as the motifs in **Fig. 01**: there is a small 1x1, 2x2 or 3x3 design, which is then rotated, so that the final motif has 4 repetitions of the initial pattern, rotated around its center. In the Az Infinitum database, many examples of 18^{th} and 19^{th} century patterns of this type can be found, such as P-18-00002 with P-18-00003, a combination of two tiles, from the times of Marquês de Pombal, which forms a 2x2 motif, or P-19-00039 (the well-known "estrela e bicha" pattern, from the 19^{th} century).

In tiling patterns of the 20^{th} century, the idea of composite motif is taken to greater lengths, see for instance (Henriques, 1998/99) for examples and analyses. We present here a few examples. The first one is the tile pattern of José de Almada Negreiros at Rua do Vale do Pereiro no. 2, Lisbon (**Fig. 06**). The pattern, which has global symmetries (rotation and translation), is of type p4. It is achieved using tiles of six types, which fit together: an empty tile, a corner turn, an arc of circle, a wave, a symmetric wave and a cross. The motif can be taken as an 8x8 square of tiles, in which case it has no symmetries. If, however, one takes as a motif a square, positioned at 45 degrees with the horizontal, with vertices at the centers of the tiles with a cross, then it has the same rotational symmetries of the square, but not the reflection symmetries (precisely due to the wavy structure of the sides). This is a remarkable fact about this pattern: that the most symmetric motif is positioned diagonally, and the lines that define the boundary of the motif are obtained by changing the sides of the square appropriately. This also happens in the famous plane tilings of M. C. Escher, in which a standard plane tiling, using squares or hexagons, is turned into a tiling with lizards of knights, by a careful deformation of the polygonal motif.

Fig. 06· Lisbon, Rua do Vale do Pereiro no. 2, tile pattern by José de Almada Negreiros, 1949, in a building by architect Pardal Monteiro (photo by José Vicente, 2013, © CML, DMC, DPC)

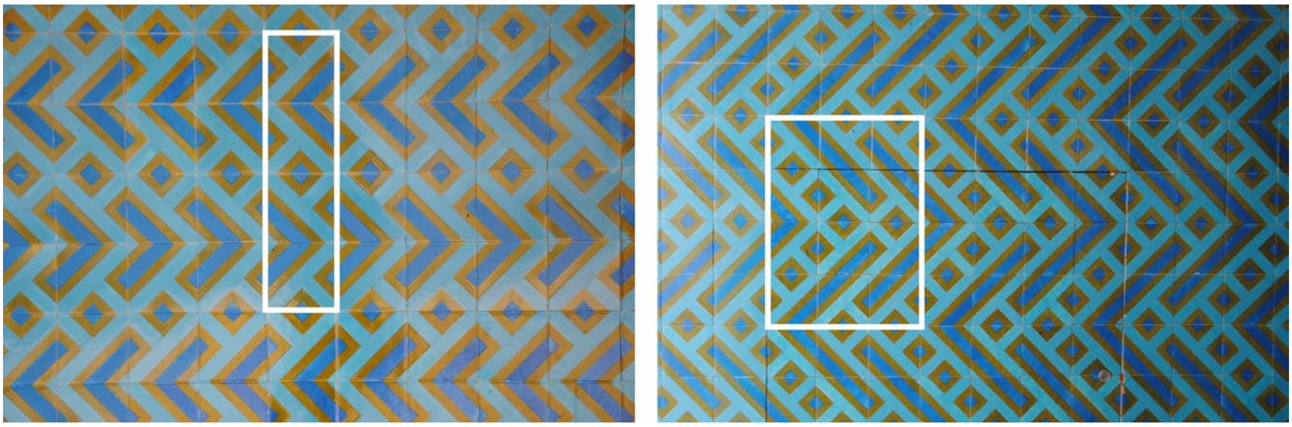

Fig. 07· Torres Vedras, Banco Nacional Ultramarino (currently Caixa Geral de Depósitos), two tile patterns by Eduardo Nery, 1971-72 (photos by Matilde Rebelo)

Another approach to composite patterns is to take one single tile and form a motif with it, but using various types of transformations, instead of the simple rotations that appear in **Fig. 01**.

Fig. 07 shows two patterns made with a single tile by Eduardo Nery, at a dependency of Caixa Geral de Depósitos in Torres Vedras (former agency of Banco Nacional Ultramarino, 1971-72). There are two more examples of use of this tile: the Mértola health center (1981) and the Contumil train station in Porto (1992-94), which has 65 panels with patterns that use this tile alone.

These patterns have composite motifs of 4x1 tiles and 4x3 tiles, respectively (marked in the figure). The tile itself has a diagonal symmetry, the motif for the first pattern has a rotation symmetry with respect to its center and the motif for the second pattern has a reflection symmetry with respect to a horizontal line. It is remarkable, thus, that both patterns have the same classification, pmg, in the system we are using. Therefore, this classification actually brought together two patterns that seemed very different at first sight, even having motifs of different sizes.

The paper (Resende, 2016) presents a more detailed analysis of patterns obtained with this tile, describing all symmetries for possible patterns with a 2x2 motif.

Ana Almeida presents more examples of Modernist pattern tiles in her Course on History of Tiles[4]. This document contains examples of tile patterns of Eduardo Nery, Querubim Lapa or Homero Gonçalves, created from a single tile, which then originates a composite pattern, following rules of their own, as in **Fig. 05**, instead of the simple rotations, as we see in **Fig. 01**. The tiles used by Querubim Lapa and Homero Gonçalves actually have no symmetries at all (except the identity), and yet, they can give rise to more than one regular pattern.

BREAKING THE PATTERN

In the mid 20th century and also in the 21st century, a different approach to pattern tiles emerged in the work of several authors. The pattern is judiciously broken, yet still keeping a sense of repetition of a motif, even though this repetition is not regular. One can use the word *quasipattern* to describe this type of work[5], just as the word quasicrystal is used in physics and crystallography to describe coverings of the plane that are *ordered but not periodic*.

4. Curso de História do Azulejo - Azulejaria Modernista, Moderna e Contemporânea. Available at http://www.museudoazulejo.gov.pt/Data/Documents/Cursos/azulejaria_2009/AA_01.pdf (2018.05.28)
5. We wish to thank Henrique Leitão for suggesting this word.

Our first example comes precisely from Maria Keil's work. Fig. 08 shows a detail of the tile panel *O mar* (the sea), located at Av. Infante Santo, Lisbon. At the bottom one can distinguish a pattern with a square motif, placed at 45 degrees, with a green triangle pointing left and a black triangle pointing right (as we move from left to right, a drawing of a shell is added to this motif). However, the panel is not a tile pattern. Unlike what happens in a tile pattern, the motif does not remain the same throughout the panel — the colors of the motif change and the motif itself is transformed by dilations. We list the dilations found in the panel.

- Vertical dilation. There are two transformations of this type: the motif is stretched by a factor of 2 and by a factor of 3 (see top left, with motif in green and blue).
- Partial vertical dilation: the motif is stretched in the vertical axis, by a factor of 2, but only in one direction (up or down), in the other direction, it is not modified. On the right side of the panel, the right half of the motif is stretched downwards and the left part upwards. In some cases a factor of 3 is used in half the motif, and brought together with a full vertical dilation, with a factor of 2, of the other half of the motif (see right side).
- Total dilation (vertical and horizontal) by a factor of 2. This appears only once, in the bottom left, and the motif is also changed to include more lines and three colors.

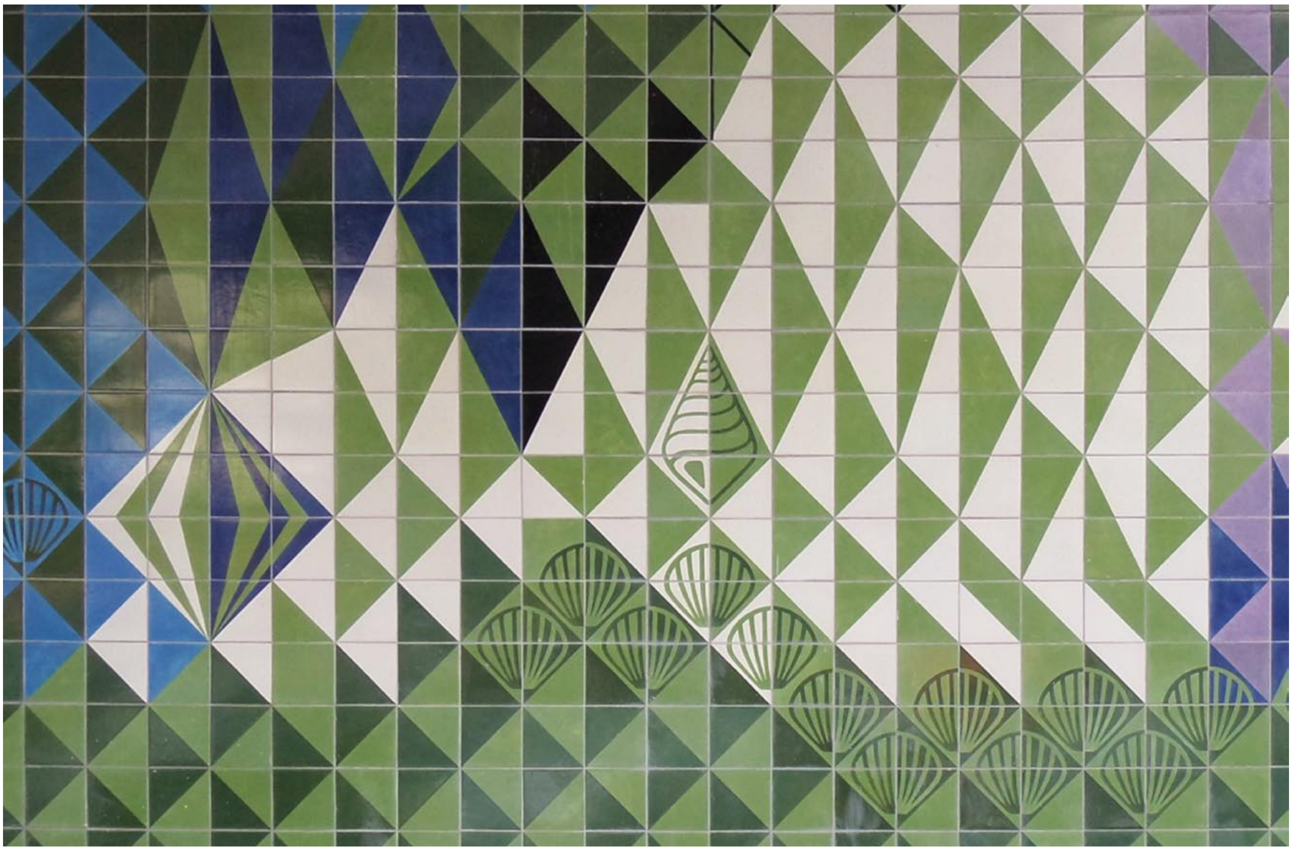

Fig. 08· Lisbon, Av. Infante Santo, detail of the panel *O Mar*, by Maria Keil, 1958-59 (photo by Pedro Freitas)

Many panels made by Maria Keil for the Lisbon Metro also present this type of dilations or reductions of a motif, which turn a simple tile pattern into a richer, non-periodic tiling, in which one can, however, still distinguish elements of a periodic tiling. This happens, for instance, in some panels located at the Metro stations of Praça de Espanha, Intendente, Picoas, Marquês de Pombal, etc. We consider the panel at Av. Infante Santo to be the richest one in this regard.

Catarina and Rita Almada Negreiros have recently created a tile panel, called *5pm*, which also uses this type of transformations (Fig. 09). For the most part of the panel, they use only one 17th century tile. Four of these tiles can be used to create the traditional "camelia" motif, using the same process as in Fig. 01: the tile is rotated, and the rotated versions are placed around a center. This can be seen in the center of Fig. 09, close to the bottom.

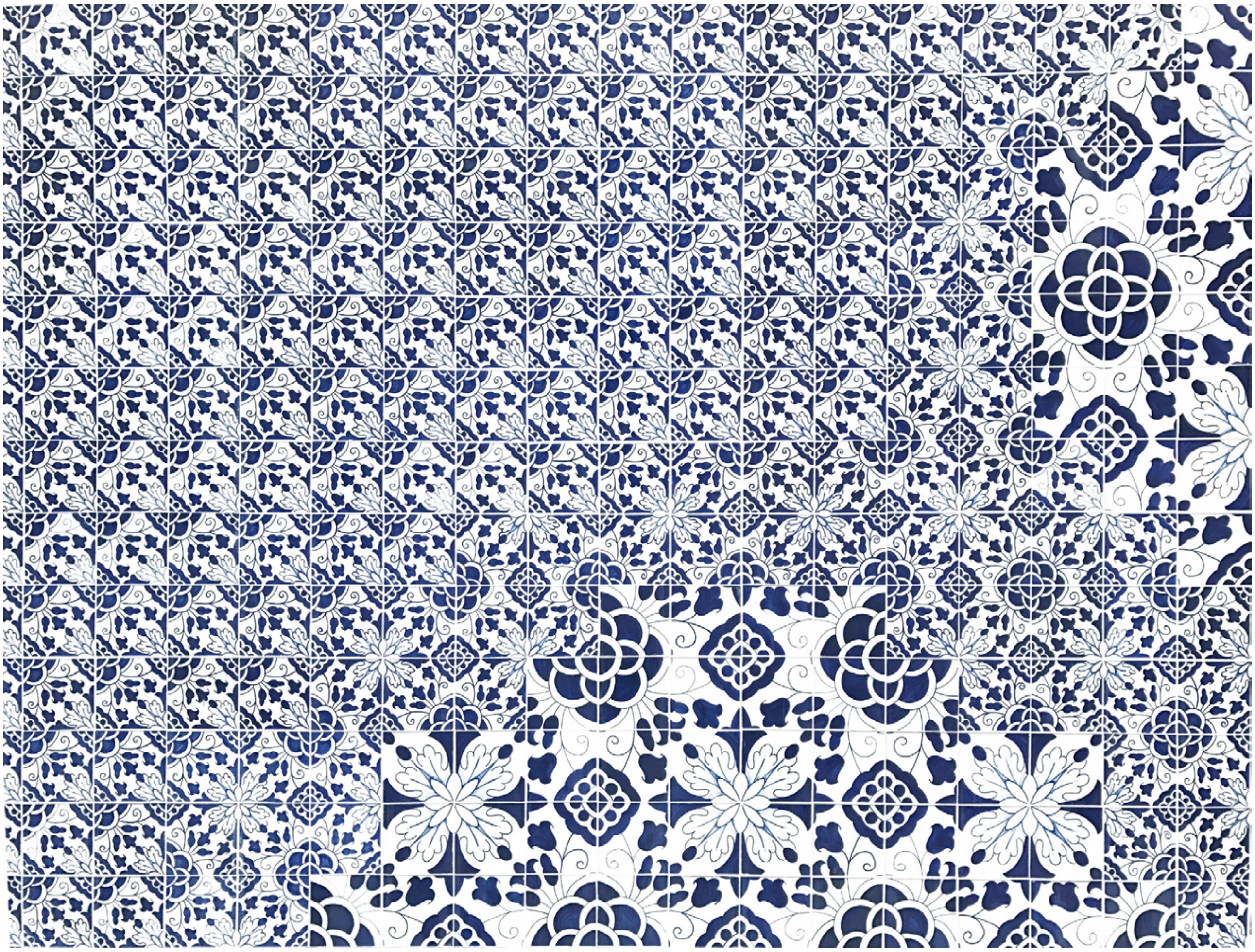

Fig. 09· Lisbon, Paço da Rainha, panel *5pm* by Catarina and Rita Almada Negreiros, 2016 (photo by Catarina and Rita Almada Negreiros)

Fig. 10· Lisbon, Av. da Índia, Casa das Galeotas, tiled wall by Catarina and Rita Almada Negreiros, 2016, and schematic drawing of the tiles used (photo by Pedro Freitas, schematic drawings by Catarina and Rita Almada Negreiros)

For most of the area of the panel, however, the tile is not rotated to form the 2x2 motif. Instead, it is repeated without change, forming a sort of background with very few symmetries (just one reflection). On some areas, this pattern is broken, and the tiles are adjusted in order to make the motif appear, covering a somewhat round area on the wall – this is what happens at the lower right part of the panel in Fig. 07. In this area, the motif is then enlarged by a factor of 2, at the center of the area, in order to make the motif more prominent. The fact that the underlying pattern has very few symmetries reinforces the impact of the symmetric enlarged motif.

This phenomenon of inclusion of dilations or reductions of pattern elements seems to be characteristic of the second half of the 20th century and of the 21st century.

Another way of breaking the pattern without losing a sense of repetition is to change the motif. We have already seen this in Fig. 06: when the motif is doubled in size, more lines are added, in order to make it more elaborate. In the Metro station *Cidade Universitária*, one panel by Maria Helena Vieira da Silva has a similar structure: the motif is just an isosceles right triangle, drawn with legs parallel to the sides of the tile, and this motif is drawn differently from one tile to the other — either changing the color, or taking away one of the sides.

A similar effect appears in the tiled walls of Casa das Galeotas (Av. da Índia, Lisbon). This house was recently rehabbed, by an architecture project of Appleton & Domingos, and the exterior tiles were redone by Catarina and Rita Almada Negreiros (Fig. 10).

The tiles originally used as a motif for these walls were blue and white square tiles, from the late 19th century, with a central motif and decorations at the four corners. At human height, the original tiles were kept, and because they were handmade, there were slight differences in hue between them, which, to the viewer, breaks the aspect of strict uniformity. In order to keep this effect, while using mechanically made tiles, a choice was made to rarify the elements in the motif, as the tiles were placed higher up on the wall: generally speaking, the higher the placement, the less elaborate the tile. The central motif was decomposed in three parts, which were gradually removed as the tiles went up, and the same happened with the corner decorations. The top tiles are completely white.

CONCLUSION

In this paper, we intended to show how the mathematical study of pattern tiles can make a valid contribution to the characterization of artistic taste and production in different times. This can be done by determination of the type of pattern or frame that is more common in a certain geographical area or period of time, in the context of what is known from Art History. In more recent works, mathematics can be used to distinguish the transformations used to reinterpret tile patterns, by showing which transformations are used to break this pattern.

In this paper we only wished to present mathematical forms of describing tile patterns (and quasipatterns) and to show, with a few examples, the type of contribution mathematics can bring to this description. There is, of course, a lot of work to be done if such a classification is to be done extensively.

We hope that mathematics can participate in the description of the identity of the tiles and the culture that produces it.

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